applied optics

Off-axis imaging with off-axis parabolic mirrors

ALBERT LIU

Condensed Matter Physics and Materials Science Division, Brookhaven National Laboratory, Upton, New York 11973-5000, USA (aliu1@bnl.gov)

Received 13 September 2023; revised 14 October 2023; accepted 20 October 2023; posted 23 October 2023; published 2 November 2023

Off-axis parabolic mirrors are commonly used to focus beams of light propagating along their optical axis. However, certain applications require the focusing of beams displaced from the optical axis. As this regime is less commonly encountered, we clarify certain unintuitive aspects of the imaging. These considerations have direct applications in implementing non-collinear optical geometries using a single parabolic mirror.

https://doi.org/10.1364/AO.505675

1. INTRODUCTION

The (off-axis) parabolic mirror is well known as a useful optical element for focusing beams at finite deflection angles. Despite the more challenging alignment compared to conventional focusing lenses, certain situations may favor, or even require, the use of parabolic mirrors. Examples of such situations may include confined optical geometries that require simultaneous beam focusing and steering, or the use of light at frequencies impractical for lenses such as terahertz radiation.

Most commonly, the use of a parabolic mirror consists of a single collimated beam of light that propagates along its optical axis, which is then focused at a finite angle upon reflection from the parabolic mirror's surface. In certain applications, however, a parabolic mirror will be used to focus a collimated beam of light that propagates parallel to, but displaced from, its optical axis. In such a situation unintuitive aspects of the optical imaging emerge, specifically after the beam has traversed the focus. Our goal in this Engineering and Laboratory Note is therefore to alert readers to these aspects when using parabolic mirrors in this way.

2. IMAGING IN A REFLECTION GEOMETRY

We first perform imaging in a reflection geometry, which for the current purpose simply involves placing a mirror at the parabolic mirror focus. The geometry of the problem is shown in Fig. 1, and we proceed through the analysis step-by-step.

A. Optical Geometry

The surface of the parabolic mirror is defined by the form $y = ax^2$, where the curvature a defines the focal length $f = \frac{1}{2a}$. We first take a (blue) incident beam at a position $x = x_0$, which results in a reflected beam from the parabolic mirror surface at a total angle 2φ, where φ is the angle formed by both the incident and reflected rays with the surface normal. The tangent of φ can be shown as

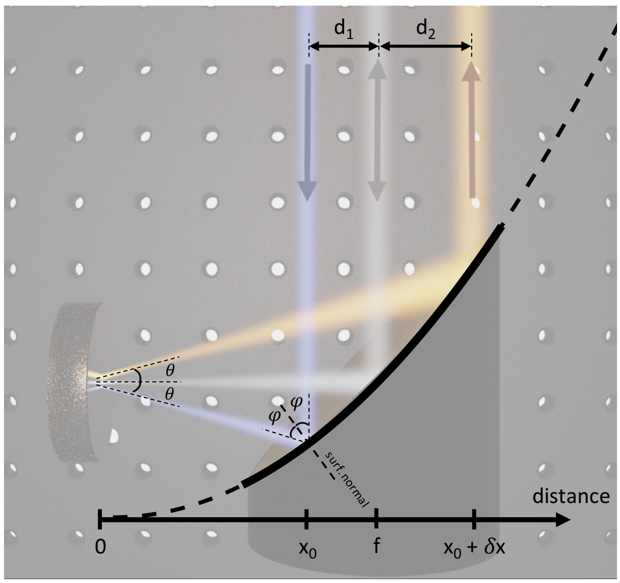

Fig. 1. Schematic of optical geometry. Background illustrates two incident beams along the optical axis (gray) and displaced from the optical axis (blue). The lateral positions of each beam are $x = f$ and $x = x_0$, respectively, where f is the focal length of the parabolic mirror. With a mirror placed at the focus position ($x = 0$), the (blue) beam at $x = x_0$ becomes a distinct reflected beam (orange) that is recollimated at $x = x_0 + \delta x$. The distances of the incident and reflected beams from the optical axis are labeled as d_1 and d_2, respectively.

$$\tan(\varphi) = \frac{a x_0^2}{\frac{1}{2} x_0} = 2 x_0 a, \tag{1}$$

which also confirms that a (gray) beam incident at $x = f = \frac{1}{2a}$ is reflected at a right angle.

After reflection from the parabolic mirror, these two rays intersect at an angle θ, defined by

$$\tan(\theta) = \frac{1}{\tan(2\varphi)} = \frac{1 - (2x_0 a)^2}{4 x_0 a}.$$

We now examine the (orange) reflected beam from the planar mirror placed at the parabolic mirror focus ($x = 0$). Given that the reflection also forms an angle θ with the planar mirror surface normal, we can form the following equality (where we define $x = x_0 + \delta x$ as the distance at which the reflected beam encounters the parabolic mirror):

$$\tan(\theta) = \frac{1 - (2x_0 a)^2}{4x_0 a} = \frac{a\,(x_0 + \delta x)^2 - \frac{1}{4a}}{x_0 + \delta x}. \tag{2}$$

This can be rearranged into a quadratic equation to solve for δx:

$$(\delta x)^2 + \left(3x_0 - \frac{1}{4x_0 a^2}\right)\delta x + \left(2x_0^2 - \frac{1}{2a^2}\right) = 0, \tag{3}$$

or in terms of the parabola focal length:

$$(\delta x)^2 + \left(3x_0 - \frac{f^2}{x_0}\right)\delta x + 2\left(x_0^2 - f^2\right) = 0. \tag{4}$$

B. Displacement from Optical Axis

We now define two distances $d_1 = f - x_0$ and $d_2 = x_0 + \delta x - f$, which represent the distances from the optical axis of the incident and reflected beams, respectively. A difference between these two distances then quantifies the change in displacement from the parabola optical axis after reflection at the focal plane. As a demonstrative case, we consider a common parabola focal length $f = 76.2$ mm. The resultant dependence of d_2 on d_1 is plotted in Fig. 2(a), which exhibits a deviation from $d_2 = d_1$ that grows more severe as d_1 increases.

C. Tangential Magnification

The difference between d_1 and d_2 shown in Fig. 2(a) also implies magnification of an optical beam and its associated ray bundle along the tangential plane. The tangential magnification, given by d_2/d_1, is plotted in Fig. 2(b) as a function of d_1. Note, however, that there is no magnification in the perpendicular direction due to symmetry of the parabolic mirror in the sagittal plane. Spatial asymmetry of the recollimated beam must therefore be accounted for in off-axis imaging in a reflection geometry.

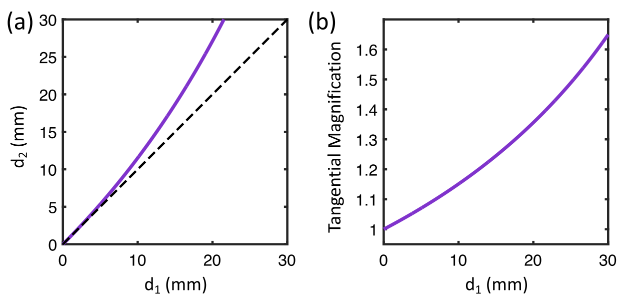

Fig. 2. (a) Relationship of the distances d_1 and d_2 from the optical axis of the incident and reflected beams, respectively, for a parabolic mirror focal length $f = 76.2$ mm and the reflection geometry illustrated in Fig. 1. As indicated by the dashed line corresponding to symmetric reflection ($d_1 = d_2$), the reflected beam is displaced from the optical axis. (b) Associated magnification in the tangential plane following recollimation in a reflection geometry. No magnification occurs in the sagittal plane.

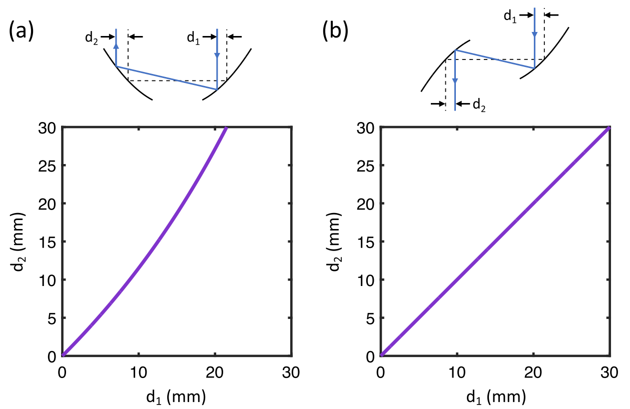

Fig. 3. Dependence of the distances d_1 and d_2 in a transmission geometry for (a) aligned and (b) opposite optical axes for the focusing and recollimation parabolic mirrors of focal lengths $f = 76.2$ mm. Asymmetric imaging is observed for the geometry in (a), but not (b).

D. Alignment and Validation

Alignment of the above off-axis geometry is a simple extension of normal alignment procedures for on-axis imaging. A reference beam along the parabolic mirror optical axis should first be established, allowing for a planar mirror to be placed at the parabola focus with an angle set to retro-reflect the reference beam. An off-axis beam at a distance d_1 or d_2 from the optical axis may then be aligned into the parabolic mirror with respect to the reference beam. In lieu of the recollimated beam profile, which is distorted due to asymmetric magnification, the alignment may be validated by comparing the displacement of the recollimated beam to the calculation detailed above.

3. IMAGING IN A TRANSMISSION GEOMETRY

As described above, displacement of a beam from the parabolic mirror optical axis and tangential magnification are unavoidable effects in a reflection geometry. However, this is not necessarily true if recollimation is performed with an additional parabolic mirror in a transmission geometry. Two geometries for the focusing and recollimation mirrors are considered in Fig. 3, which exhibit distinct behaviors.

In Fig. 3(a), the optical axes of the two parabolas are aligned with respect to each other. This can be recognized as an unfolded version of the reflection geometry in Fig. 1, and identical displacement of the recollimated beam is found. In Fig. 3(b), however, where the optical axes of the two mirrors are rotated by 180° with respect to each other, no such displacement of the recollimated beam occurs. This geometry is therefore advantageous for preserving symmetry in the optical system, although certain conditions may favor the geometry in Fig. 3(a) due to optical aberrations [1–3].

4. CONCLUSION

Here we have described, from the perspective of geometric optics, unintuitive aspects of off-axis imaging with off-axis parabolic mirrors. Specifically, displacement of an optical beam following recollimation and asymmetric magnification are effects that result. Though unavoidable in a reflection geometry, these effects may be mitigated in a transmission geometry with appropriate orientation of the collimating mirror.

The present discussion is particularly relevant for implementing spectroscopic experiments in non-collinear phase-matching geometries with a single parabolic focusing mirror. Such a simple experimental geometry is especially beneficial at mid-infrared and terahertz frequencies [4], where conventional lenses do not suffice and cumbersome optical setups involving multiple focusing parabolas are often used. One such experiment implementing multidimensional terahertz spectroscopy [5] in a non-collinear geometry using a single parabolic mirror has been demonstrated recently [6].

Disclosures. The author declares no conflicts of interest.

Data availability. No data were generated or analyzed in the presented research.

REFERENCES

1. P. Arguijo and M. S. Scholl, "Exact ray-trace beam for an off-axis paraboloid surface," Appl. Opt. **42**, 3284–3289 (2003).
2. C. Brückner, G. Notni, and A. Tünnermann, "Optimal arrangement of 90° off-axis parabolic mirrors in THz setups," Optik **121**, 113–119 (2010).
3. N. J. Laurita, B. Cheng, R. Barkhouser, *et al.*, "A modified 8f geometry with reduced optical aberrations for improved time domain terahertz spectroscopy," J. Infrared Millim. Terahertz Waves **37**, 894–902 (2016).
4. D. Nicoletti and A. Cavalleri, "Nonlinear light–matter interaction at terahertz frequencies," Adv. Opt. Photonics **8**, 401–464 (2016).
5. J. Lu, X. Li, Y. Zhang, *et al.*, "Two-dimensional spectroscopy at terahertz frequencies," Top. Curr. Chem. **376**, 6 (2018).
6. A. Liu, D. Pavicevic, M. H. Michael, *et al.*, "Probing inhomogeneous cuprate superconductivity by terahertz Josephson echo spectroscopy," arXiv, arXiv.2308.14849 (2023).